Closing Tues: HW 10.1

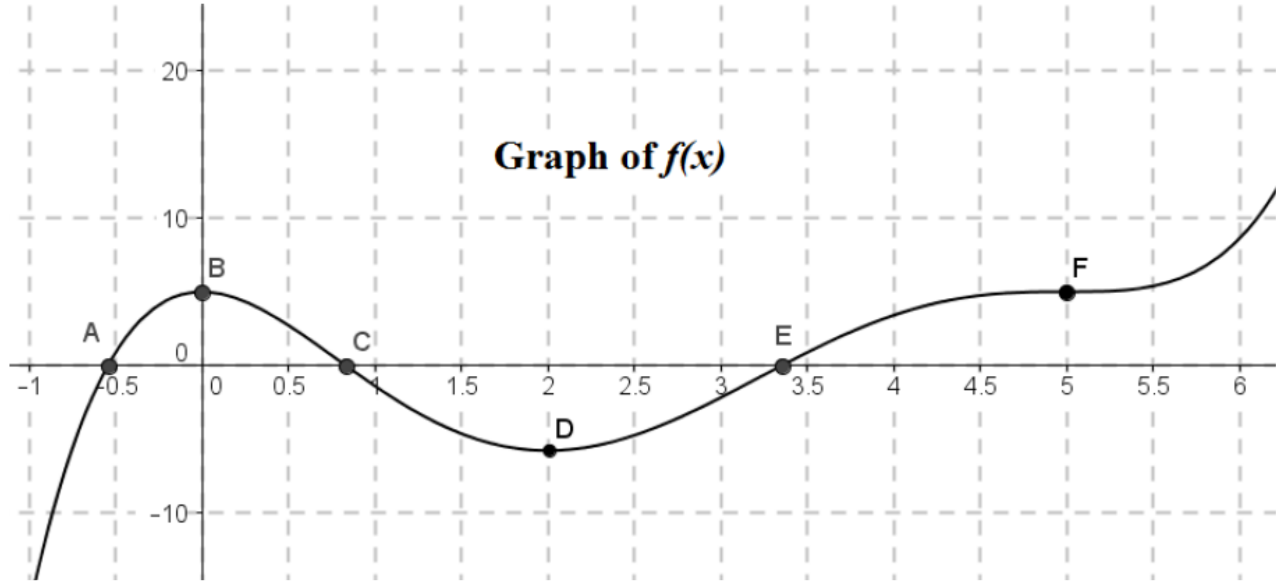
Closing Thurs: HW 10.2

Exam 1 will be returned Tues

Entry Task (directly from HW)
Consider $P(t) = 33t + 6t^2 - t^3$.
For what value of t is $P(t)$ increasing?
(You'll need a calculator to get some decimals). Also do the full 1st deriv. number line analysis that we did in lecture on Friday.

10.2 Concavity

Consider the given $y = f(x)$ graph (same graph from last lecture). Draw the tangent line at each point. Is the tangent line above or below the curve near that point?



Terminology:

If $f''(x)$ is *positive* at $x = a$,
then $f(x)$ is **concave up** at $x = a$.
This means the tangent slopes are increasing near $x = a$ and the tangent line is below the graph at $x = a$.

If $f''(x)$ is *negative* at $x = a$,
then $f(x)$ is **concave down** at $x = a$.
This means the tangent slopes are decreasing near $x = a$ and the tangent line is above the graph at $x = a$.

If $f''(x) = 0$ at $x = a$, then we say $x = a$ is a ***possible point of inflection***.

A ***point of inflection*** is any point where the concavity *changes*.

Example:

Let $f(x) = \frac{1}{2}x^4 - 3x^2 + 5x + 1$

Find all intervals when $f(x)$ is concave up and find all inflection points.

Summary of 1st and 2nd deriv. facts

$f(x)$	$f'(x)$	$f''(x)$
horiz. tangent	zero	
increasing	positive	
decreasing	negative	
possible inflection	hor. tangent	zero
concave up	increasing	positive
concave down	decreasing	negative

1st Deriv Analysis:(to find critical points, increasing, decreasing, local max/min, h.p.o.i)

Step 1: Critical Points

Find $f'(x)$ and solve $f'(x) = 0$.

Step 2: Draw number line. Between critical points, pick values of x and plug into $f'(x)$ to see if it is positive or negative.

Step 3: Make appropriate conclusions.

2st Deriv Analysis: (to find inflection points, concave up/down)

Step 1: Possible Inflection Points

Find $f''(x)$ and solve $f''(x) = 0$.

Step 2: Draw number line. Between possible infection points, pick values of x and plug into $f''(x)$ to see if it is positive or negative.

Step 3: Make appropriate conclusions.

Example:

Let $g(x) = x^3$.

Find all local optima and points of inflection, then sketch the graph.

Example: Let $TC(q) = 5000q^2 + 125000$ dollars for producing q things.

Recall: Overall average cost per item is given by

$$AC(q) = \frac{TC(q)}{q} = \frac{5000q^2 + 125000}{q}$$

Analyze AC(q).

(What does it look like?, what are relative max/min? etc….)